



## **Simplified Whisker Risk Model Extensions**

### **1. Extensions to Whisker Risk Model**

The whisker risk Monte Carlo model described in the prior SERDEP work (Ref. 1) was extended to incorporate the following:

- Parallel flat plates
- Parallel round pins
- Perpendicular flat plates

#### **1.1 Whisker Mirror**

The model extensions used both the whisker mirror described in prior work and explicit modeling of the surfaces for the parallel configurations. The perpendicular flat plate model used explicit modeling of both surfaces and did not implement the whisker mirror.

## 2. Whisker Risk Data Subset

The general approach for developing the simplified whisker risk model was to develop generic relationships based on view factors and spacing distributions developed from specific geometries as described in Table 1 through Table 2. Spacing and pitch dimensions are as follows:

- Parallel plate – spacing is the spacing between the plates
- Parallel round pins – pitch is distance between centerlines
- Perpendicular plate – spacing (S) is the spacing between the center of the plates (see Figure 1)

Monte Carlo modeling was conducted for the listed geometries for uncoated leads only.

**Table 1 – Dimensions Considered – Parallel Flat Plate Model**

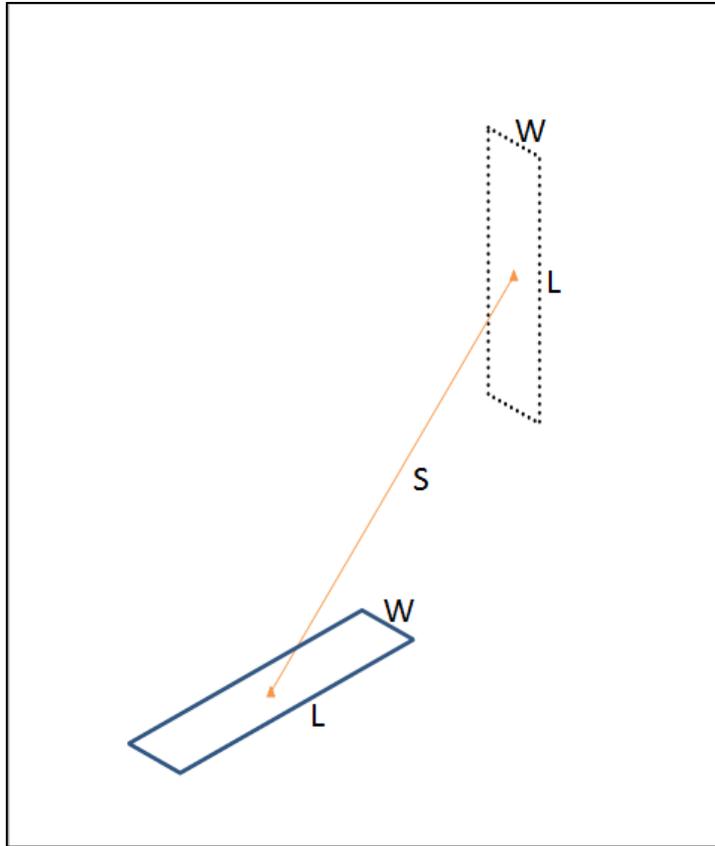
<b>Length (mm)</b>	<b>Width (mm)</b>	<b>Spacing (mm)</b>
0.3378	0.127	0.2
0.65	0.127	0.2
0.65	0.127	0.5
1	0.1	0.1
1	0.1	0.2
1	0.2	0.1
1	0.2	0.2
2	0.1	0.1
2	0.1	0.2
2	0.2	0.1
2	0.2	0.2
0.5	0.1	0.1
0.5	0.1	0.2
0.5	0.2	0.1
0.5	0.2	0.2
0.0635	0.14986	0.33
0.6812	0.14986	0.33
0.6812	0.14986	0.7366

**Table 2 – Dimensions Considered – Parallel Round Lead Model**

<b>Length (mm)</b>	<b>Diam. (mm)</b>	<b>Pitch (mm)</b>
0.3378	0.127	0.8
0.65	0.127	0.8
0.65	0.127	0.8
1	0.1	1
1	0.1	2
1	0.2	1
1	0.2	2
2	0.1	1
2	0.1	2
2	0.2	1
2	0.2	2
0.5	0.1	1
0.5	0.1	2
0.5	0.2	1
0.5	0.2	2
0.0635	0.14986	1.27
0.6812	0.14986	1.27
0.6812	0.14986	1.27

**Table 3 – Dimensions Considered – Perpendicular Flat Plate Model**

<b>Length (L,mm)</b>	<b>Width (W, mm)</b>	<b>Spacing (S, mm)</b>
0.3378	0.127	0.804546
0.65	0.127	1.025305
0.65	0.127	1.025305
1	0.1	1.414214
1	0.1	2.12132
1	0.2	1.414214
1	0.2	2.12132
2	0.1	2.12132
2	0.1	2.828427
2	0.2	2.12132
2	0.2	2.828427
0.5	0.1	1.06066
0.5	0.1	1.767767
0.5	0.2	1.06066
0.5	0.2	1.767767
0.0635	0.14986	0.942927
0.6812	0.14986	1.379707
0.6812	0.14986	1.379707
0.3378	0.127	0.380282
0.65	0.127	0.601041
0.65	0.127	0.813173
1	0.1	0.777817
1	0.1	0.848528
1	0.2	0.777817
1	0.2	0.848528
2	0.1	1.484924
2	0.1	1.555635
2	0.2	1.484924
2	0.2	1.555635
0.5	0.1	0.424264
0.5	0.1	0.494975
0.5	0.2	0.424264
0.5	0.2	0.494975
0.0635	0.14986	0.278247
0.6812	0.14986	0.715026
0.6812	0.14986	1.002536



**Figure 1 – Perpendicular Plate Spacing**

### 3. Simplified Model Development

Development of the simplified model was based on developing whisker view factor and spacing distribution based on lead and pad dimensions. All calculations were based on the use of dimensionless parameters to avoid complications in the simplified model due to selection of units.

#### 3.1 Whisker Shape Factor

The whisker shape factor represents the probability that an infinite whisker will bridge between adjacent leads. A trial and error method in conjunction with the Excel solver was used to develop a metric that has best correlation to whisker shape factor based on dimensional calculations easily derived from part/pad data.

##### 3.1.1 Lead Whisker Parallel Plates and Pins

The shape factor for parallel plates and pins is based on the same metric ( $M_L$ ) as used in the prior work (Ref. 2) with the understanding that the single side area ( $A_s$ ) is the projected area along the pitch axis (for plates  $A_w=A_s$ ) and the lead thickness is the plate width or pin diameter:

$s$  = Lead Spacing

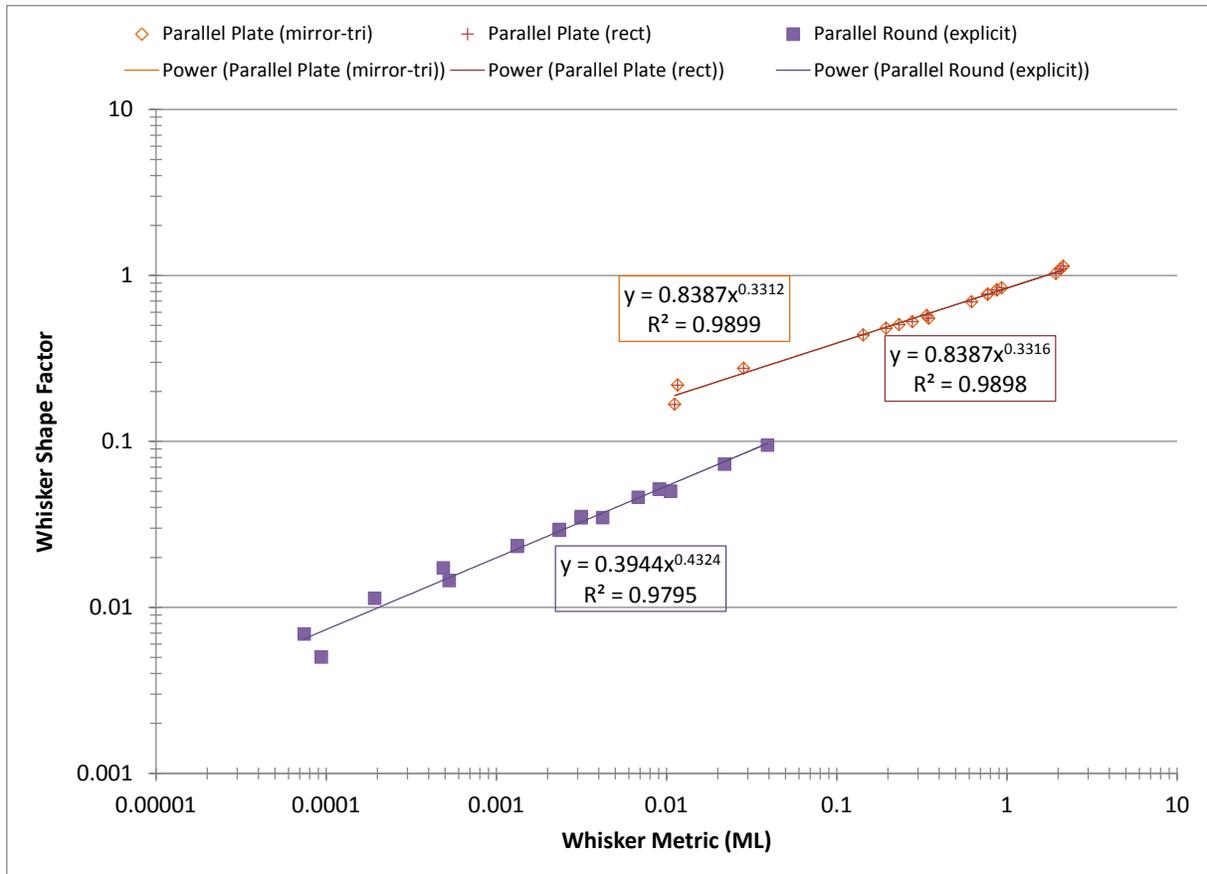
$t$  = Lead Thickness

$A_w$  = Whiskerable Area

$A_s$  = Single Side Area

$$M_L = \left( \frac{A_s}{A_w + s^2} \right)^{1.421215} \cdot \left( \frac{t}{s} \right)^{1.153123}$$

A plot providing the modeled shape factor as a function of the above metric and the derived equation is provided in Figure 1.



**Figure 2 – Parallel Plate and Round Lead Shape Factor Correlation**

### 3.1.2 Perpendicular Plate Whisker Shape Factor

The shape factor for perpendicular plates is given by the following metric ( $M_L$ ) with the spacing as shown in:

$s$  = Plate Spacing (see text)

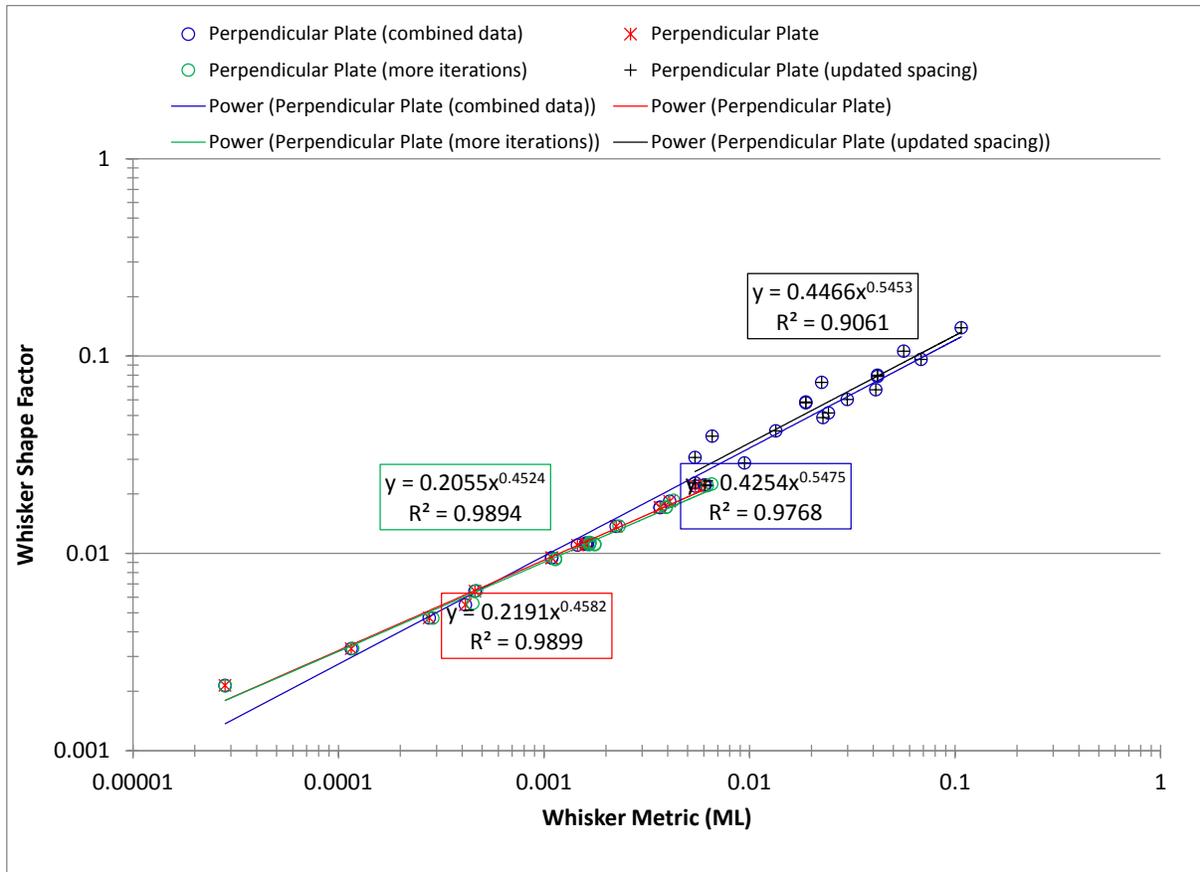
$t$  = Plate Width

$A_w$  = Whiskerable Area

$A_s$  = Projected Side Area Perpendicular to Surface =  $A_w$

$$M_L = \left( \frac{A_s}{A_w + s^2} \right)^{2.473507905} \cdot \left( \frac{t}{s} \right)^{-0.418031594}$$

A plot providing the modeled shape factor as a function of the above metric and the derived equation is provided in Figure 2.



**Figure 3 – Perpendicular Plate Shape Factor Correlation**

### 3.2 Spacing Distribution

The spacing distribution provides a cumulative fraction versus length between the minimum and maximum spacing. Development of an appropriate distribution requires calculation of the minimum and maximum spacing and the appropriate intermediate fractions.

#### 3.2.1 Minimum Spacing

The minimum spacing is determined by the specific geometry as follows:

- Parallel flat plates – the spacing between the plates
- Round leads – the pitch minus the lead diameter
- Perpendicular plates – the center spacing minus the lead length divided by the square root of 2

#### 3.2.2 Maximum Spacing

The maximum lead spacing is determined by the specific geometry as the extreme distance between the far corners of the geometry as follows:

- Parallel flat plates – square root of sum of squares of plate width, length, and spacing
- Round leads – square root of sum of squares of lead diameter, lead length, and pitch

- Perpendicular plates – use the following equation:

$$s_{\max} = \sqrt{\frac{l^2}{2} + \sqrt{2} \cdot l \cdot s + s^2 + w^2}$$

l = Length  
w = Width  
s = Spacing

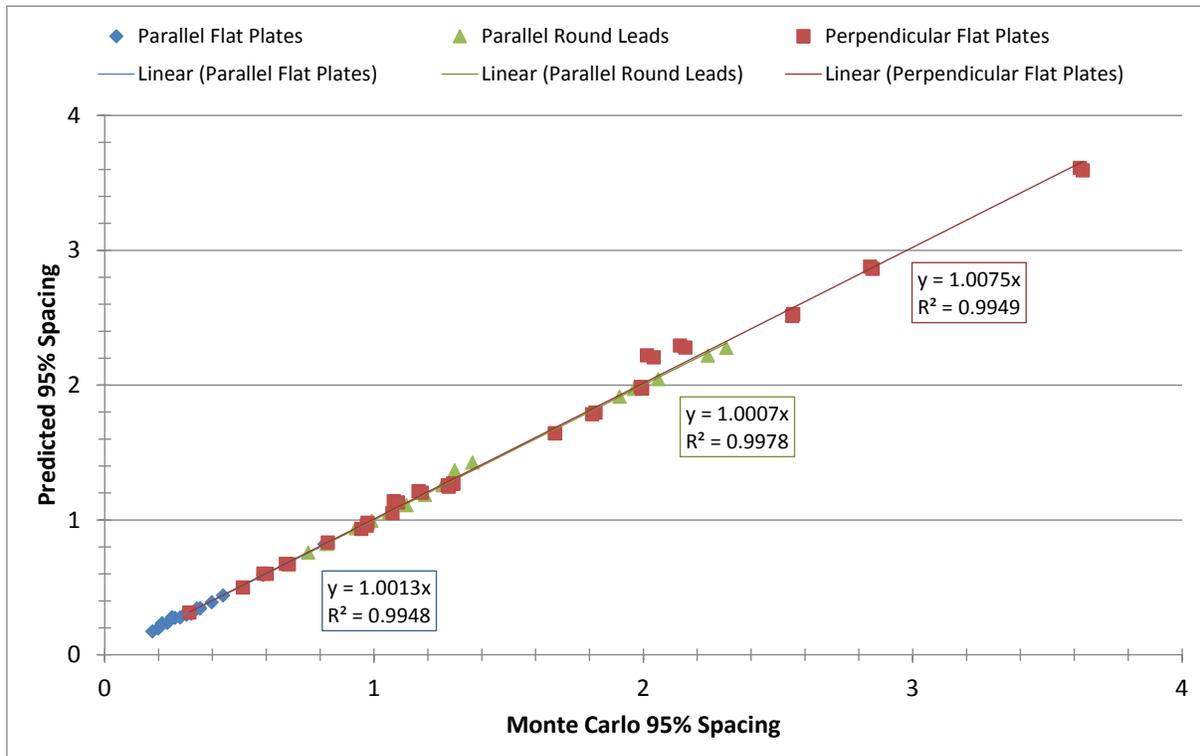
### 3.2.3 Distribution Development

#### 3.2.3.1 0%, 95%, and 100% Distribution Points

Because of the controlled nature of the parallel and perpendicular geometries, the 0% distribution point is the minimum spacing and the 100% point is the maximum spacing. The intermediate points are based on the 95% point as calculated by a linear best fit based on the length, width, and minimum and maximum spacing. Specific coefficients for the 95% length prediction are shown in Table 4. Correlation between the Monte Carlo spacing and predicted are shown in Figure 4.

**Table 4 – Coefficients for Length, Width, and Max./Min. Spacing for 95% Spacing**

	Length	Width	Spacing	
			Min.	Max.
Parallel Flat Plates	0.387985	0.457803	1.144546	-0.35014
Perpendicular Flat Plates	-0.33047	-0.202	0.034695	0.999461
Parallel Round Leads	0.017016	0.635506	0.037845	0.364181



**Figure 4 – Correlation for 95% Spacing Prediction**

### 3.2.3.2 Intermediate Distribution Points

The distributions are developed based on a non-dimensional distribution by subtracting the minimum spacing from the spacing values, dividing this value by the difference between 95% spacing point described in paragraph 3.2.3.1 and the spacing as follows (Note: this is a different approach from that used in prior work, Ref. 2):

$$S = \frac{(s_i - s_{\min})}{(s_{95\%} - s_{\min})}$$

$S$  = Non-Dimensional Spacing

$s_i$  = Spacing of Point (from Monte Carlo)

$s_{95\%}$  = 95% Spacing (see text)

$s_{\min}$  = Minimum Spacing

This produces a value of zero at 0% and nominally one at 95%. Plotting the results as a function of cumulative spacing fraction produces reasonably consistent results as shown in Figure 5 through Figure 7.

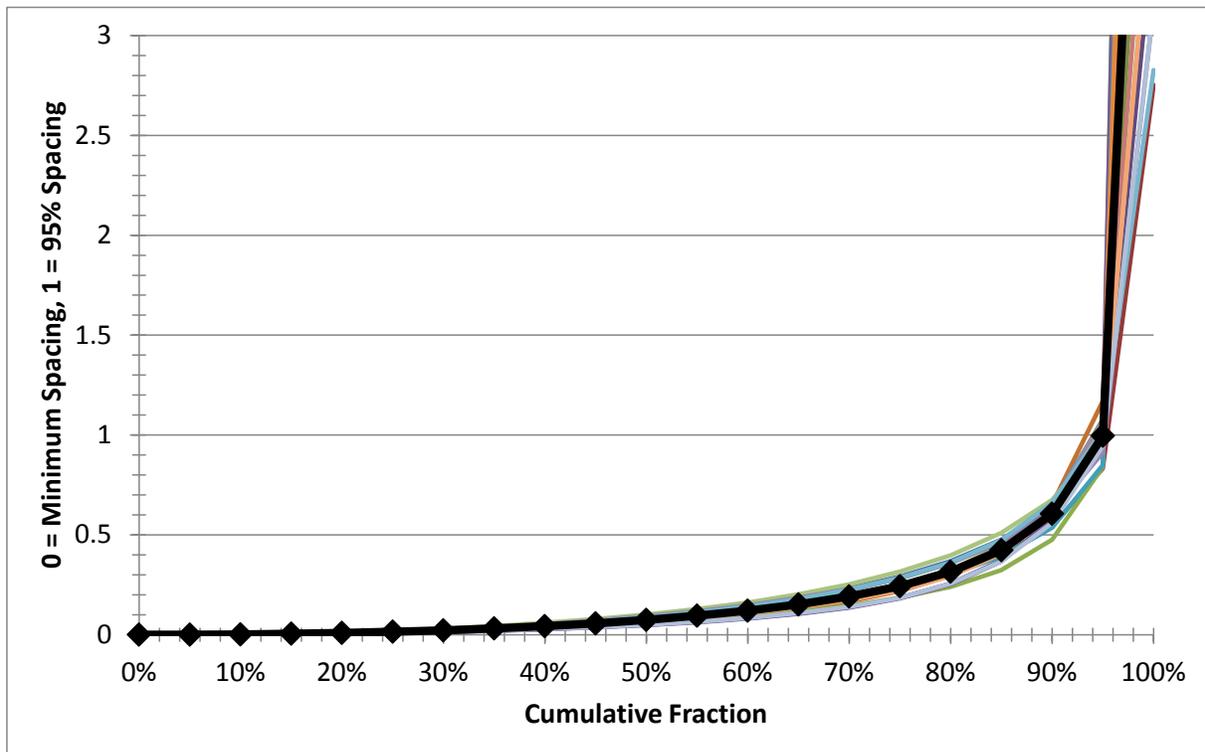


Figure 5 – Non-Dimensional Spacing Distribution for Parallel Flat Plates

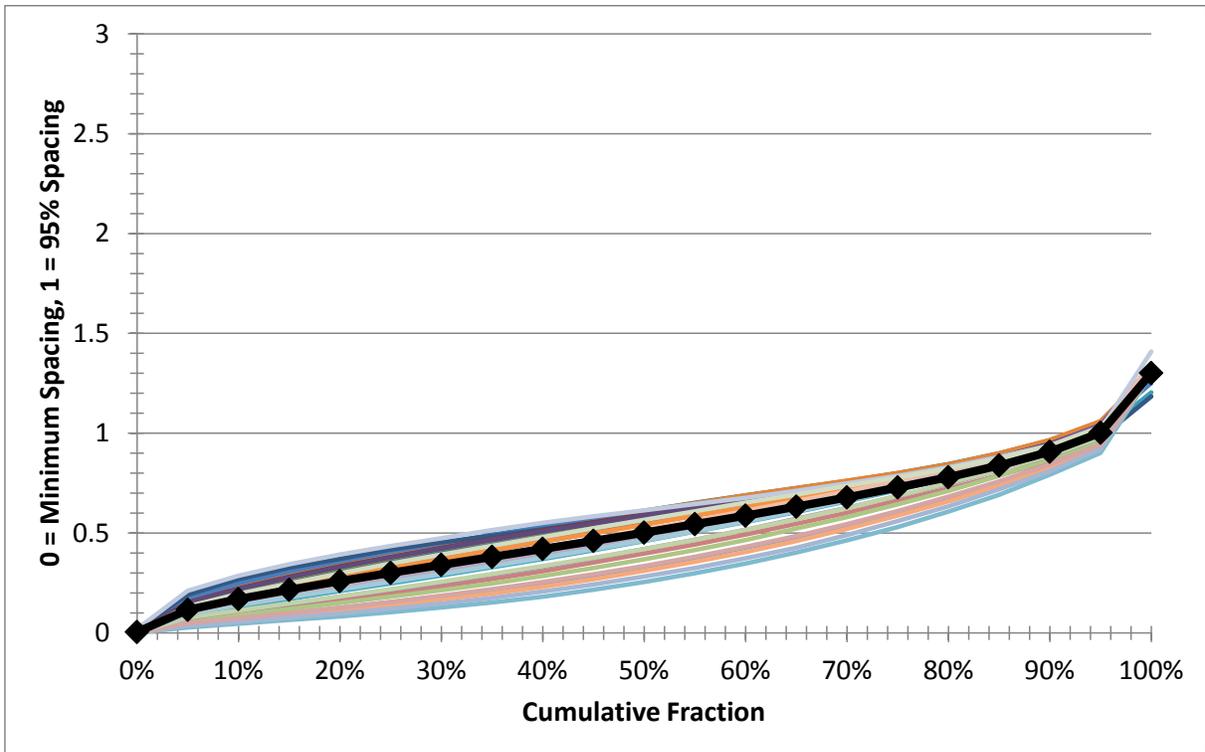


Figure 6 – Non-Dimensional Spacing Distribution for Perpendicular Flat Plates

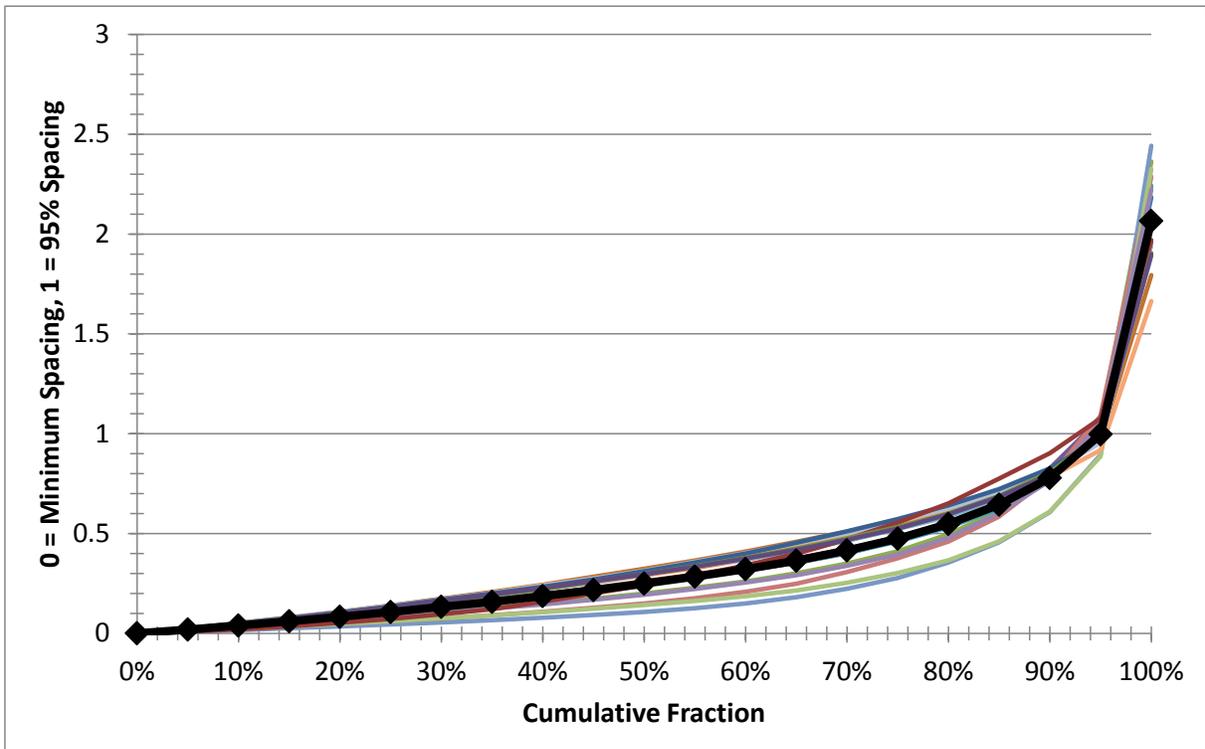


Figure 7 – Non-Dimensional Spacing Distribution for Parallel Round Leads

Specific non-dimensional distribution values for each type of scenario are summarized in Table 5. These values are used in conjunction with the appropriate minimum, 95%, and maximum spacings to extract the specific length as a function of cumulative fraction.

**Table 5 – Non-Dimensional Whisker Distribution**  
(0 = minimum spacing, 1 = 95% spacing)

Cumulative	Parallel Plate	Perpendicular Plate	Parallel Round Lead
0%	0.0000	0.0000	0.0000
5%	0.0005	0.1151	0.0188
10%	0.0022	0.1694	0.0391
15%	0.0051	0.2162	0.0606
20%	0.0093	0.2593	0.0831
25%	0.0150	0.3006	0.1072
30%	0.0224	0.3410	0.1326
35%	0.0317	0.3807	0.1585
40%	0.0432	0.4206	0.1871
45%	0.0572	0.4610	0.2172
50%	0.0743	0.5024	0.2490
55%	0.0950	0.5444	0.2840
60%	0.1204	0.5876	0.3220
65%	0.1519	0.6324	0.3650
70%	0.1916	0.6789	0.4155
75%	0.2434	0.7279	0.4748
80%	0.3154	0.7800	0.5483
85%	0.4234	0.8385	0.6449
90%	0.6056	0.9078	0.7786
95%	1.0000	1.0000	1.0000
100%	Use maximum from text		

### 3.3 Special Case of Parallel Large and Small Plates

A closed form solution for round plate with an infinitesimal source was used to develop an approximate view factor and distribution relationship for parallel large and small rectangular plates with only one plate generating whiskers. The radius for the large plate was approximated based on the same area as the rectangular target plate. Development of the analytical solution and inspection of Monte Carlo results leads the preliminary hypothesis that whiskering between two unequal areas is reciprocal in that the number of whiskers making contact does not depend on which area generates the whiskers. It also follows that the distribution fractions will be identical for whiskers sourced from either plate. Using the geometry shown in Figure 8, the following view factor relationship was developed:

### View Factor - Point to Large Circular Plate

$$F_V = \frac{s}{\pi^2} \int_0^r \left[ \frac{2 \cdot \pi \cdot \rho}{(\rho^2 + s^2) \cdot \rho} \right] d\rho$$

$$F_V = \frac{2 \cdot \text{atan}\left(\frac{r}{s}\right)}{\pi}$$

$r$  = Radius of Plate

$s$  = Spacing

$F_V$  = View Factor

### Equating Areas

$$\pi \cdot r^2 = l_s \cdot w_s$$

$$r = \sqrt{\frac{l \cdot w}{\pi}}$$

$l$  = Length of Target Plate

$w$  = Width of Target Plate

$$F_V = \frac{2 \cdot \text{atan}\left(\frac{\sqrt{l \cdot w}}{\sqrt{\pi \cdot s}}\right)}{\pi}$$

For Whiskers Sourced at the Large Plate and Hitting Small Plate:

$$F_{VL} = F_V \cdot \frac{A_S}{A_L}$$

$A_S$  = Area of Small Plate

$A_L$  = Area of Large Plate

$F_{VL}$  = View Factor for Whiskers Originating at Large Plate

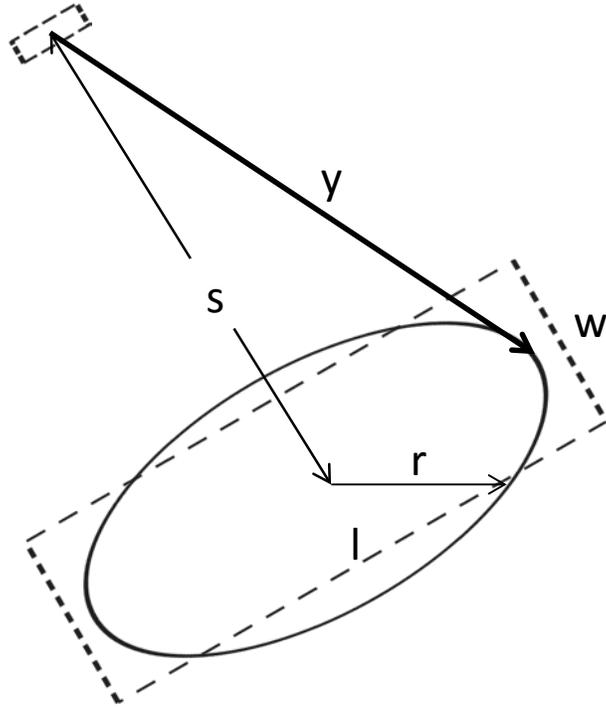


Figure 8 – Small and Large Plate Approximation

Table 6 – Comparison of Approximate (closed-form) and Monte Carlo View Factor

Configuration	Plate Length	Plate Width	Plate Spacing	Target Length	Target Width	View Factor		
						Equation	Monte Carlo	Error
Plate to QFP lead + solder (0.5mm space)	9	7	0.5	1.6	1	0.0236	0.0236	-0.01%
Plate to 0603 lead + solder (0.5mm space)	9	7	0.5	1	0.75	0.0111	0.0110	0.15%
Plate to QFP lead + solder (1mm space)	9	7	1	1.6	1	0.0218	0.0217	0.76%
Plate to 0603 lead + solder (1mm space)	9	7	1	1	0.75	0.0102	0.0102	0.61%
Plate to QFP lead + solder (2mm space)	9	7	2	1.6	1	0.0186	0.0184	1.05%
Plate to 0603 lead + solder (2mm space)	9	7	2	1	0.75	0.0087	0.0086	1.51%
Plate to QFP lead + solder (4mm space)	9	7	4	1.6	1	0.0136	0.0133	2.25%
Plate to 0603 lead + solder (4mm space)	9	7	4	1	0.75	0.0064	0.0063	1.07%

The above relationships provide good agreement with the Monte Carlo results (see Table 6). The view factor relationships can be used to develop the spacing distribution relationship, as follows:

### Cumulative Spacing Distribution Fraction

$$F_S = \frac{F_r}{F_v}$$

$$r = \sqrt{y^2 - s^2}$$

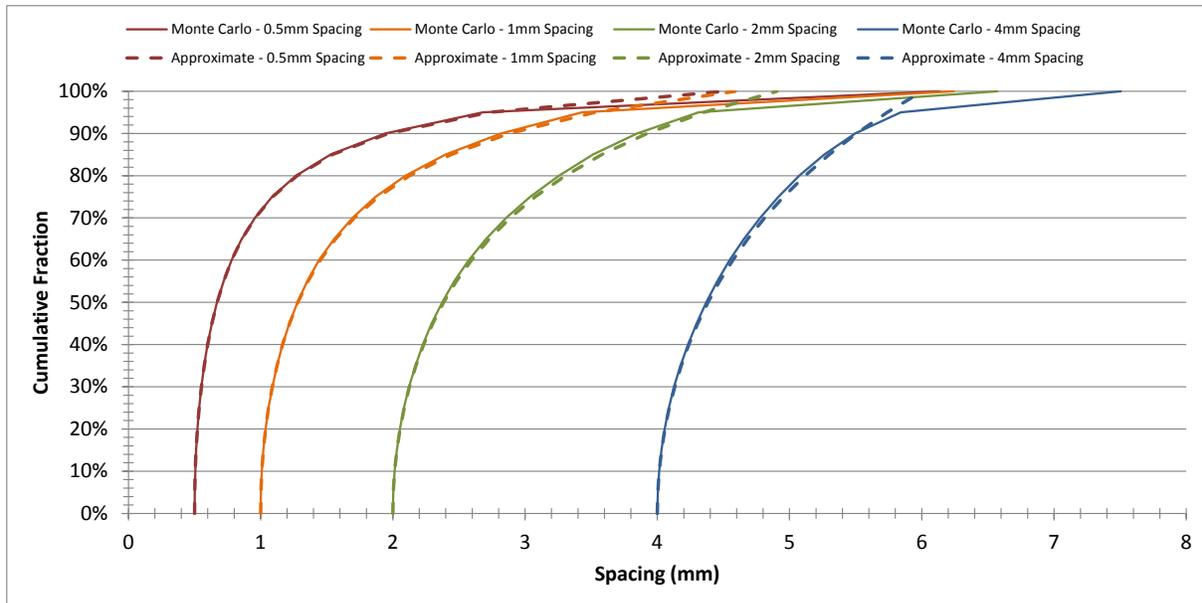
$$F_S = \frac{\text{atan}\left(\frac{\sqrt{y^2 - s^2}}{s}\right)}{\text{atan}\left(\frac{\sqrt{l \cdot w}}{\sqrt{\pi \cdot s}}\right)}$$

$F_r$  = View Factor at Radius  $r$

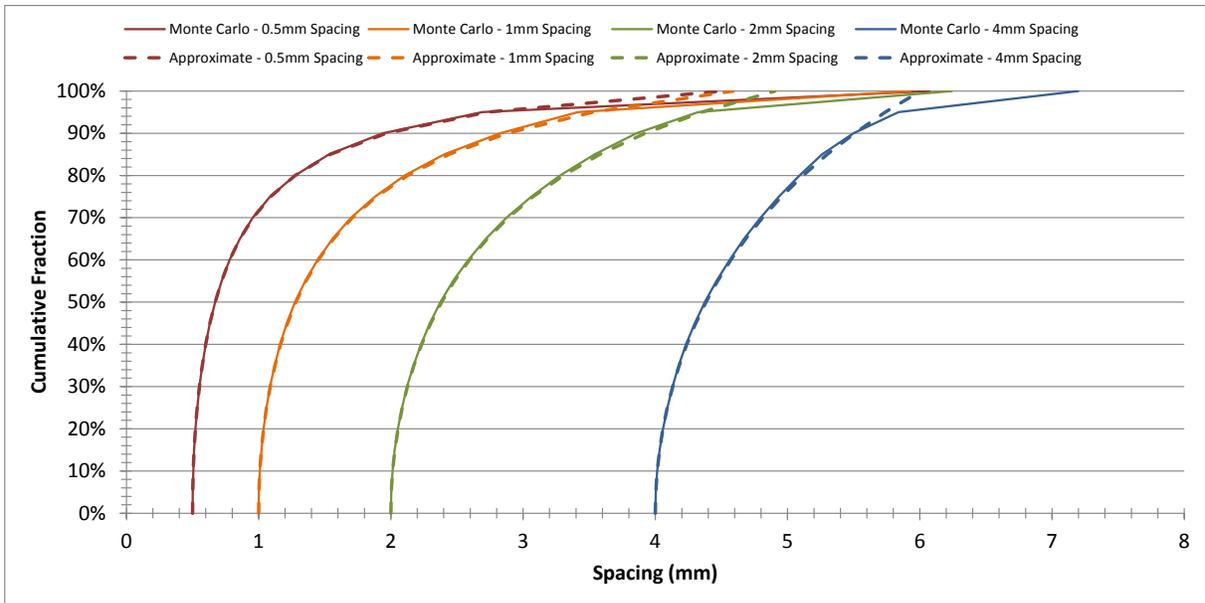
$y$  = Whisker Length

$F_S$  = Whisker Cumulative Fraction

Comparison of the approximate and Monte Carlo distribution factors are shown in Figure 9 and Figure 10. Divergence between the approximate and Monte Carlo models at 100% is due to the difference between the maximum from the use of the circle approximation and the actual maximum. Use of the approximate relationship is conservative since the 100% point represents a shorter length, a less conservative but valid approach would be to use the actual calculated maximum based on the geometry.



**Figure 9 – Comparison of Whisker Length Distributions for Plate to QFP Lead & Solder**



**Figure 10 – Comparison of Whisker Length Distribution for Plate to 0603 Terminal & Solder**

## 4. Simplified Whisker Risk Model Spreadsheet

### 4.1 General Information

Green highlighted cells are for normal user input, blue highlighted cells are for default values that can but normally do not need to be changed. Values in red font are not recommended to change. There is no system of units but all units need to be consistent including part and pad dimensions and whisker length distributions. Calculated values are shown in a blue font. Rows and columns not necessary for use of the spreadsheet are hidden. There are eight tabs included in the spreadsheet:

- User Notes
- Roll Up
- Model
- Whisker
- Distribution Plots
- Shorting Prob.
- Revision Info.
- Reference Data

### 4.2 User Notes Tab

The **User Notes** tab provides for general user notes and are not used in the calculations.

### 4.3 Roll Up Tab

The Roll Up Tab (see Figure 11) allows for input of varying dimensions and model cases and calculates the total number of shorts. To use the roll up capability, blank out cells with a red border on the **Model** tab, the first row of data (red border) is then used for calculations in the **Model** tab.

	Case (# parallel, Number 2-perpendicular, 3-round, 4-large/small)	Case Description	Length	Blk./Width	Pitch/Spacing	Smaller Length (case 4 only)	Smaller Width (case 4 only)	Shorts per Instance	Total Shorts
10	Case 1 5 1	Equal Parallel Plates	9	7	1			625.3448	3115.724202
11	Case 2 1 2	Equal Perpendicular Plates	8	5	30			0.000543	0.000642899
12	Case 3 1 3	Parallel Round Leads	0.65	0.127	0.5			0.569932	0.56993227
13	Case 4 1 4	Large & Small Parallel Plates	9	7	0.5	1.6	1	30.90509	30.90509688

Figure 11 – Roll Up Tab

## 4.4 Model Tab

The **Model** tab (see Figure 9) provides the definition of the geometry, applied voltage, and the results.

### 4.4.1 Data Entry

If dimensions have been entered roll up above, specific lead dimensions are calculated and provided with a default value (in parentheses). The specific data to be entered is as follows:

- Description (optional, Case 4)
- Case Code (4)
- Length (9)
- Width (7)
- Spacing (0.5)
- Smaller Length (1.6) – unequal plate model only (case 4)
- Smaller Width (1) – unequal plate model only (case 4)

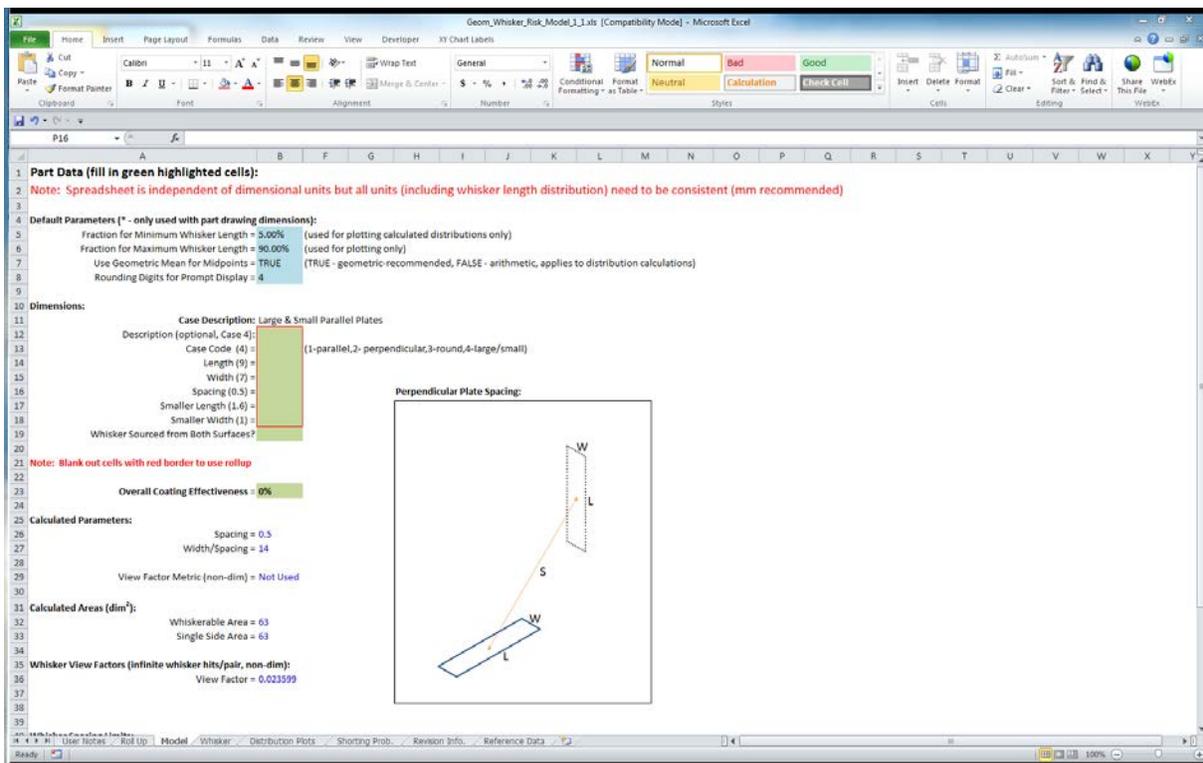


Figure 12 – Upper Portion of Model Tab

### 4.4.2 Other Defining Information

In addition to the dimensional information described above, an option that applies for only case 4 (unequal parallel plates) is provided that allows whiskers to be sourced from one or both surfaces. The default is TRUE indicating whiskers can originate at either of the unequal surfaces, change this to FALSE for whiskers originating at only one surface. For other cases, whiskers can always originate at both surfaces. The coating effectiveness represents the fraction of the target lead covered with conformal coating.

### 4.4.3 Applied Voltage and Final Results

The applied voltage and results are located at the lower portion of the **Model** tab. The applied voltage is used in conjunction with the appropriate shorting probability to determine the number of bridges that result in shorts. The result, expressed as the total number of shorts for the part is highlighted in light red.

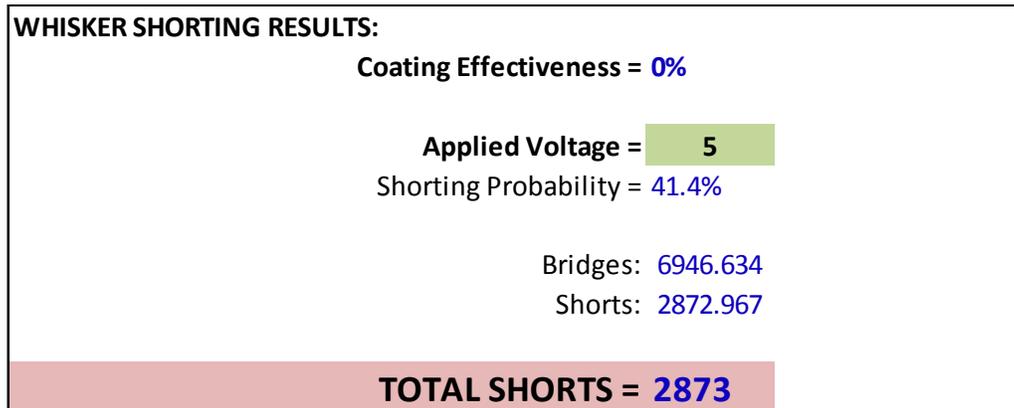


Figure 13 – Lower Portion of Model Tab

### 4.4.4 Default Parameters

The default parameters are highlighted in blue (see Figure 9) and may not need to be changed by the user except with a change in the system of units. Specific parameters that can be set are:

- Fraction for Minimum Whisker Length – this value is used for plotting calculated distributions
- Fraction for Maximum Whisker Length – this value is used to limit the numerical whisker length for plotting
- Use Geometric Mean for Midpoints – TRUE – use geometric mean for distribution calculations (recommended), FALSE – use arithmetic mean
- Rounding Digits for Prompt Display – sets how many digits are displayed in prompts with values

## 4.5 Whisker Tab

The whisker tab (see Figure 13) defines the appropriate whisker length distribution and density to be applied to the appropriate spacing distributions. The whisker density is defined on this tab (cell B6) based on the number of whiskers generated per square unit (based on square of units used in the rest of the spreadsheet, so if mm used it is per mm<sup>2</sup>). Cell B4 is a cell used to select the distribution type for each tab (1-numerical, 2-lognormal, 3-log Cauchy, 4-Cauchy, 5-Weibull).

### 4.5.1 Calculated Distributions

The calculated distributions are selected by values of 2 through 5 in cell B4. The following parameters need to be supplied to define the distribution:

- Fraction for Short Whisker
- Fraction for Long Whisker
- Minimum Length (if applicable)
- Short% length
- Long% length

The distribution parameters are calculated based on lengths for user-defined short and long cumulative fractions. If the specific model parameters are known, they can be entered in cells B22 through B24 instead of calculating from the cumulative percentages.

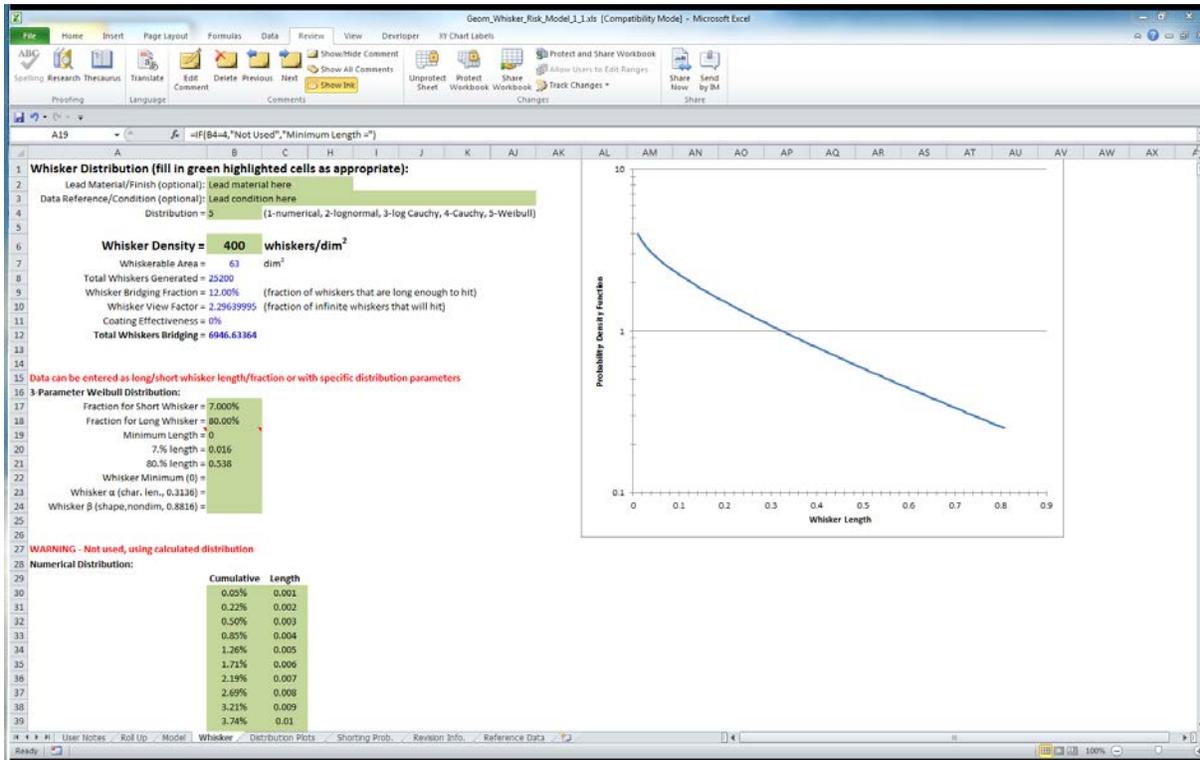


Figure 14 – Whisker Tab

#### 4.5.2 Numerical Distribution

The numerical distribution is selected by entering 1 in cell B4. The numerical distribution consists of an ordered list of cumulative percentages and corresponding whisker lengths (see Figure 14) for up to 100 data points. Start data entry in cells B30 and C30 and fill in as many data points as needed, clear the remaining cells beyond those needed to define the distribution. Warnings will be generated if the minimum length is less than the minimum whisker spacing or if the maximum length is insufficient to calculate the bridging probability. It should be noted that the minimum/maximum lengths are the average of the first/last two lengths in the table. A warning will also be generated above the numerical distribution if a lognormal distribution is selected. Units for length column need to be in units consistent with the rest of the spreadsheet.

Whisker\_Risk\_Model\_1\_2.xlsx

	A	B	C	H	I
26	<b>Numerical Distribution:</b>				
27		<b>Cumulative</b>	<b>Length</b>		
28		1.70%	0.01		
29		19.99%	0.03		
30		40.23%	0.05		
31		55.73%	0.07		
32		66.88%	0.09		
33		74.87%	0.11		
34		80.64%	0.13		
35		88.04%	0.17		
36		92.26%	0.21		
37		94.80%	0.25		
38		96.40%	0.29		
39		97.44%	0.33		
40		98.13%	0.37		
41		98.62%	0.41		
42		98.96%	0.45		
43		99.20%	0.49		
44		99.38%	0.53		
45		99.51%	0.57		
46		99.61%	0.61		
47		99.69%	0.65		
48		99.75%	0.69		
49		99.80%	0.73		
50		99.93%	0.95		
51		99.97%	1.17		
52		99.99%	1.39		
53		99.99%	1.61		
54		100.00%	1.83		
55		100.00%	2.05		
56		100.00%	2.27		
57		100.00%	2.49		
58		100.00%	2.71		
59		100.00%	2.93		
60		100.00%	3.15		
61		100.00%	3.37		
62		100.00%	3.59		
63		100.00%	3.92		
64		100.00%	4.14		
65		100.00%	4.36		
66		100.00%	4.58		
67		100.00%	4.8		
68		100.00%	5.02		
69		100.00%	5.24		
70		100.00%	5.46		
71		100.00%	5.68		

Part Data Lead Whisker Solde

**Figure 15 – Numerical Whisker Length Distribution**

## 4.6 Distribution Plots Tab

The **Distribution Plots** tab (see Figure 15) has no inputs but provides the following probability density plots:

- Spacing distributions as calculated by the spreadsheet (solid lines)
- Lead whisker length distributions as defined by the whisker length definition tab (dashed lines)
- Solder bridging probability density as calculated by the spreadsheet (double solid lines)

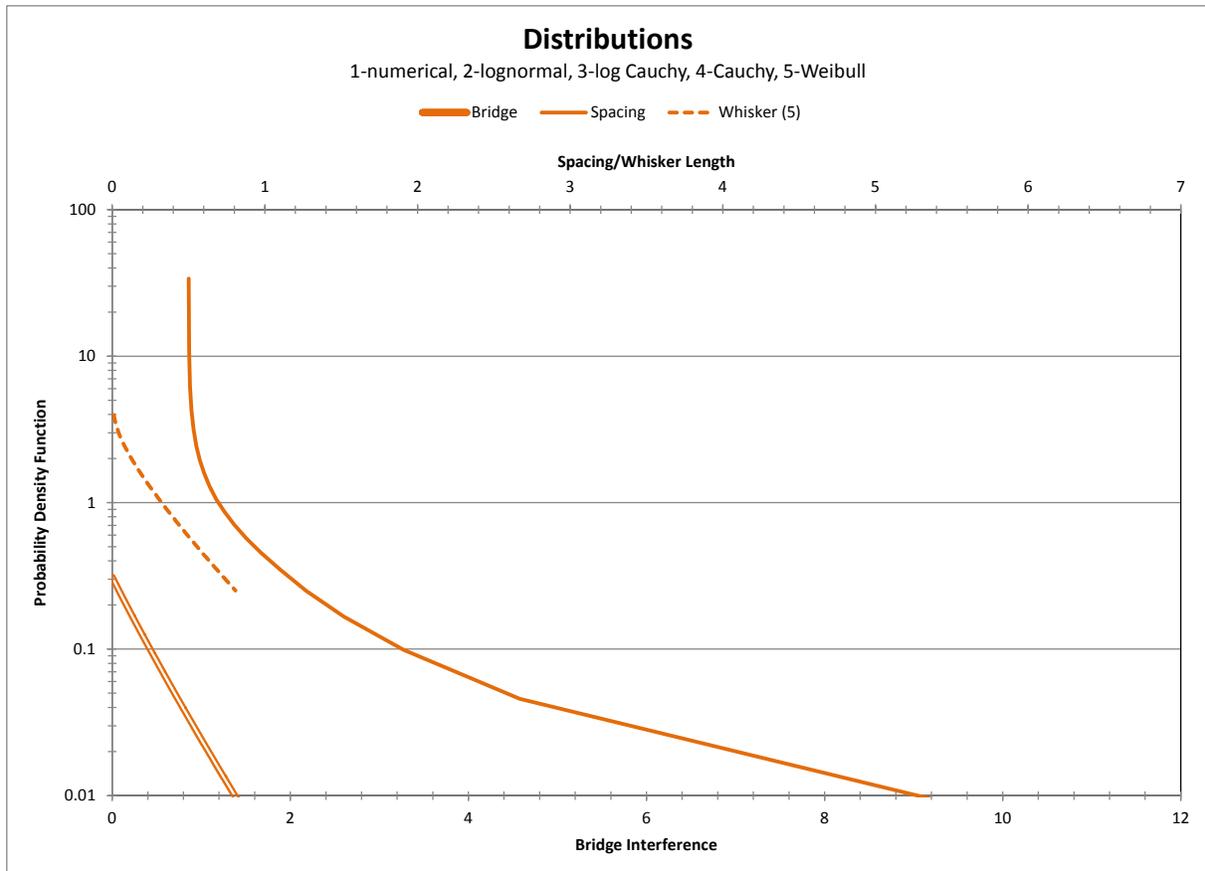


Figure 16 – Distribution Plots

## 4.7 Shorting Prob. Tab

The **Shorting Prob.** tab (see Figure 16) establishes the probability that a bridged whisker would result in a short at a given voltage using a lognormal distribution. The tab has inputs for the lognormal parameters to establish the shorting distribution with initial data based on published results from Courey (see Ref. 3).

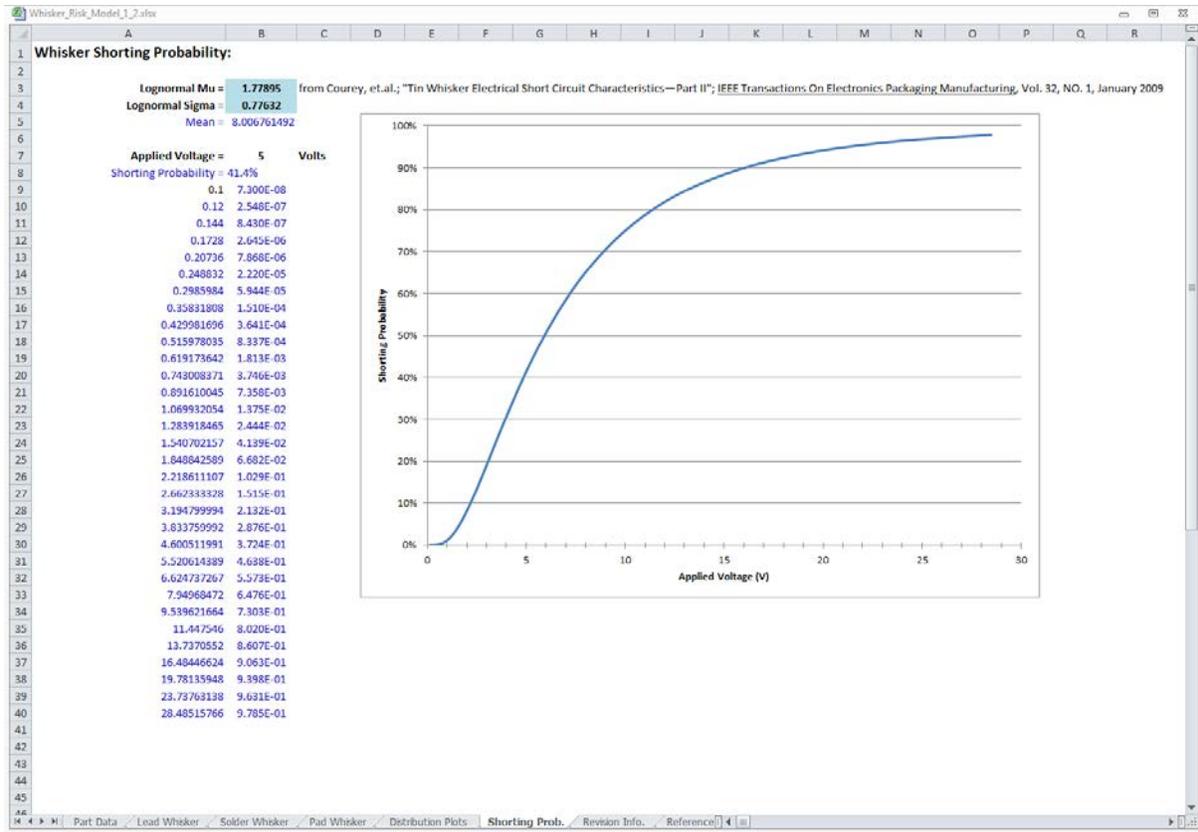


Figure 17 – Shorting Probability Tab

#### 4.8 Revision Info. Tab

The Revision Info. tab provides a log of changes to the whisker risk spreadsheet with revision number and date.

#### 4.9 Reference Data Tab

The Reference Data tab provides a summary of the specific cases used to establish the whisker modeling correlations. Dimensions are as shown in Table 1 through Table 3 and are included in the spreadsheet for completeness. Caution should be used for cases that represent significant extrapolation from listed component types.

## 5. References

1. Meschter, Snugovsky, Kennedy, Bagheri, Kosiba; "SERDP Tin Whisker Testing and Modeling: High Temperature/High Humidity (HTHH) Conditions"; Defense Manufacturers Conference (DMC) December 2-5, 2013 Orlando, Florida
2. 2014 paper
3. Panashchenko, Lyudmyla; "Evaluation of Environmental Tests for Tin Whisker Assessment"; University of Maryland, Master's thesis 2009
4. Courey, et.al.; "Tin Whisker Electrical Short Circuit Characteristics—Part II"; IEEE Transactions on Electronics Packaging Manufacturing, Vol. 32, NO. 1, January 2009